SOLUTION

Phy201

Electricity & Magnetism

Final Exam, Fall 2007

1 February 2008

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NAME	**ID#**

8 problems, 32 points (2 bonus point)
120 minutes of allocated time

Good Luck to All

1. Two identical beads each have a mass m and charge q. When placed in a hemispherical bowl of radius R with frictionless, nonconducting walls, the beads move, and at equilibrium they are a distance R apart (Fig. P23.68). Determine the charge on each bead.

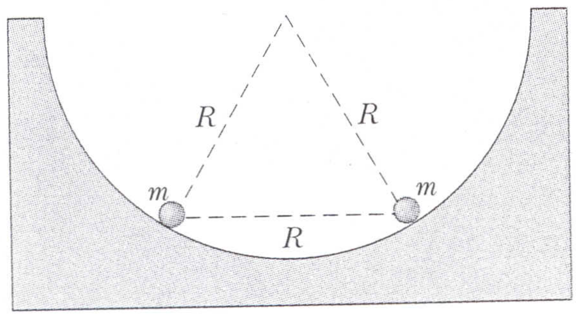
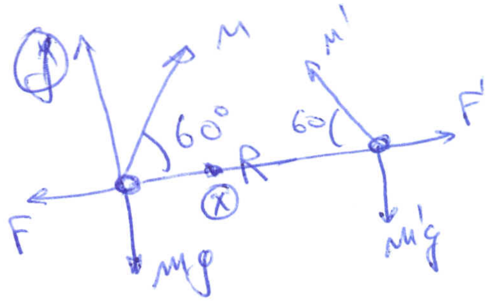


Figure P23.68



$\sum F_y$ (1) $= n \sin 60° - mg = 0$

$\therefore\ n = \dfrac{mg}{\sin 60°}$

$\sum F_x$ (1) $= -F_e + n \cos 60 = 0$

$F_e = \dfrac{kq^2}{R^2} = n\cos 60 \Rightarrow \dfrac{kq^2}{R^2} = \dfrac{mg}{Tg\,60°}$

$\dfrac{kq^2}{R^2} =$ (1) $\dfrac{mg}{\sqrt{3}}$

$\therefore\ \boxed{q = R\sqrt{\dfrac{mg}{k\sqrt{3}}}}$ (1)

2. A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c, as shown in Figure P24.55. (a) Find the magnitude of the electric field in the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$. (b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

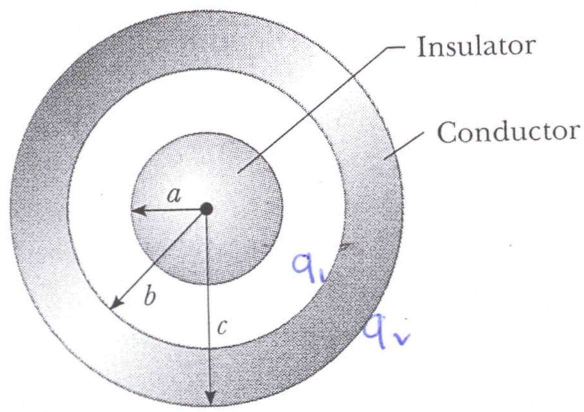


Figure P24.55

a) $\oint \vec{E}\cdot d\vec{A} = E(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$ (Gauss' Law)

$r<a$ $\quad q_{in} = \rho\left(\frac{4}{3}\pi r^3\right) \Rightarrow E = \frac{q_{in}}{4\pi r^2 \epsilon_0} \Rightarrow$

$E = \frac{\rho r}{3\epsilon_0}$

$a<r<b$ and $c<r$ $\quad q_{in} = Q \Rightarrow E = \frac{Q}{4\pi r^2 \epsilon_0}$

$b<r<c \Rightarrow E=0$ since $q_{in}=0$

b) q_1 = induced charge on inner surface

for $b<r<c$ $\Rightarrow q_1 + Q = 0$ and $\sigma_1 = \frac{q_1}{4\pi b^2} = -\frac{Q}{4\pi b^2}$

q_2 = induced charge on outside surface of Hollow sphere

$\therefore q_1 + q_2 = 0 \Rightarrow \sigma_2 = \frac{q_1}{4\pi c^2} = \frac{Q}{4\pi c^2}$

3. A wire of finite length that has a uniform linear charge density λ is bent into the shape shown in Figure P25.46. Find the electric potential at point O.

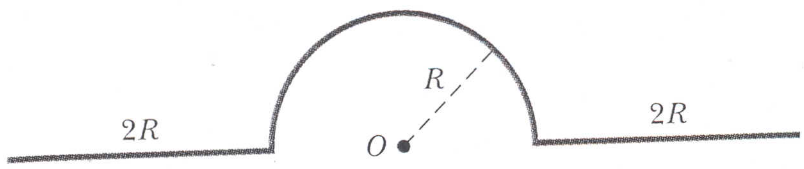


Figure P25.46

$$V = k\int \frac{dq}{r}$$

$$= k\int_{-3R}^{R} \frac{\lambda\,dx}{-x} + k\int_{s.c} \frac{\lambda\,ds}{R} + k\int_{R}^{3R} \frac{\lambda\,dx}{x}$$

$$= -k\lambda \ln(-x)\Big|_{-3R}^{R} + \frac{k\lambda}{R}\pi R + k\lambda \ln(x)\Big|_{R}^{3R}$$

$$= k\lambda \ln\frac{3R}{R} + k\lambda\pi + k\lambda \ln 3$$

$$\boxed{V = 2k\lambda \ln 3 + k\lambda\pi}$$

4. A vertical parallel-plate capacitor is half filled with a dielectric for which the dielectric constant is 2.00 (Fig. P26.71a). When this capacitor is positioned horizontally, what fraction of it should be filled with the same dielectric (Fig. P26.71b) so that the two capacitors have equal capacitance?

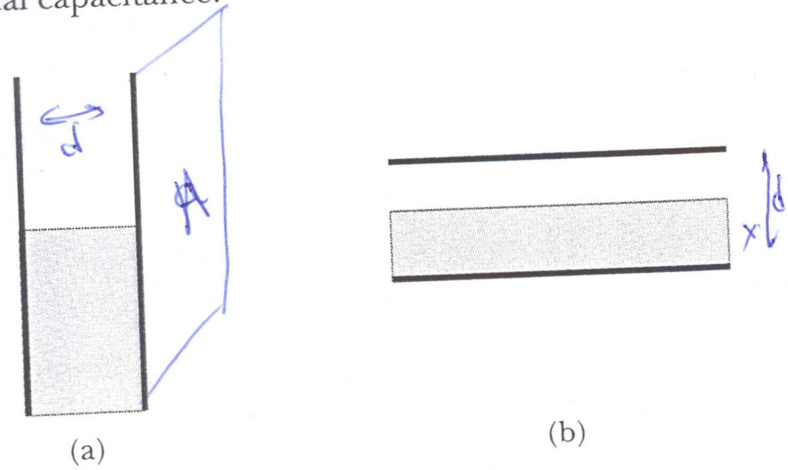


Figure P26.71

$\kappa^2 - \kappa^2 f + f = \kappa$

$f(1-\kappa^2) = \kappa - \kappa^2$

$f = \dfrac{\kappa - \kappa^2}{1-\kappa^2}$

$f = \dfrac{2-4}{1-4} = \dfrac{-2}{-3}$

$f = \dfrac{2}{3}$

a) is for 2 capacitors in //

b) is " " " in series.

f is fraction filled or $\frac{x}{d}$

$a \Rightarrow C_{(a)eq} = \dfrac{\epsilon_0 A/2}{d} + \dfrac{\kappa \epsilon_0 A/2}{d} = \left(\dfrac{\kappa+1}{2}\right)\dfrac{\epsilon_0 A}{d}$

$b \Rightarrow C_{(b)eq} \Rightarrow \dfrac{1}{C_{eq}} = \dfrac{fd}{\kappa\epsilon_0 A} + \dfrac{(1-f)d}{\epsilon_0 A} = \left[\dfrac{f+\kappa(1-f)}{\kappa}\right]\dfrac{d}{\epsilon_0 A} \Rightarrow \left[\dfrac{\kappa}{f+\kappa(1-f)}\right]\dfrac{\epsilon_0 A}{d} = C_{(b)eq}$

$\dfrac{2}{f+2(1-f)}\dfrac{\epsilon_0 A}{d}$

$\left(\dfrac{2}{2-f}\right)\dfrac{\epsilon_0 A}{d}$

$C_{(a)} = C_{(b)}$ $\left(\dfrac{\kappa+1}{2}\right)\dfrac{\epsilon_0 A}{d} = \dfrac{\kappa}{f+\kappa(1-f)}\dfrac{\epsilon_0 A}{d}$

$(\kappa+1)[f+\kappa(1-f)] = 2\kappa$

$\kappa f + \kappa^2(1-f) + f + \kappa(1-f) = 2\kappa$

$\kappa f + \kappa^2 - \kappa^2 f + f + \kappa - \kappa f = 2\kappa$

$\kappa^2(1-f) + f = \kappa$

5. One lightbulb is marked "25 W 120 V," and another "100 W 120 V"; this means that each bulb converts its respective power when plugged into a constant 120-V potential difference. (a) Find the resistance of each bulb. (b) How long does it take for 1.00 C to pass through the dim bulb? How is this charge different at the time of its exit compared with the time of its entry? (c) How long does it take for 1.00 J to pass through the dim bulb? How is this energy different at the time of its exit compared with the time of its entry? (d) Find the cost of running the dim bulb continuously for 30.0 days if the electric company sells its product at $0.070 0 per kWh. What product *does* the electric company sell? What is its price for one SI unit of this quantity?

a) $I = \frac{\Delta U}{R}$; $P = (\Delta U) I = \frac{(\Delta U)^2}{R} \Rightarrow$

$R_1 = \frac{(\Delta U)^2}{P} = \frac{(120)^2}{25} = \boxed{576\,\Omega}$ $\quad R_2 = \frac{(120)^2}{100} = \boxed{144\,\Omega}$

b) $I = \frac{P}{\Delta V} = \frac{25}{120} = 0.208\,A = \frac{Q}{t} = \frac{1C}{t}$

$t = \frac{1}{0.208} = \boxed{4.8\,s = t}$ (charge has lower potential energy)

c) $P = 25\,W = \frac{\Delta U}{t} = \frac{1J}{t} \Rightarrow$

$t = \frac{1}{25} = \boxed{0.04\,s}$ (energy changes from electric to heat and light)

d) $\Delta U = Pt = 25 * 86400 * 30 = 64.8 \times 10^6\,J$

Cost $64.8 \times 10^6 \times \left(\frac{0.07}{1000}\right) * \frac{1}{3600} = 1.26\,\$$

Cost / joule $= \frac{0.07}{3.6 \times 10^6} = \boxed{\$\,1.94 \times 10^{-8} / \text{joule}}$

6. The circuit shown in Figure P28.33 has been connected for a long time. (a) What is the voltage across the capacitor? (b) If the battery is disconnected, how long does it take the capacitor to discharge to one-tenth its initial voltage?

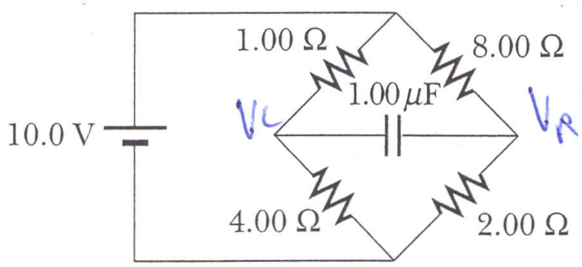


Figure P28.33

a) $V_L = 8V$ because of voltage divider

$I_L = \frac{10}{9} = 2A$ $\quad V_L = 10 - 2\times 1 = 8V$

$I_R = \frac{10}{10} = 1A$ $\quad V_R = \left(\frac{2}{2+8}\right) \times 10 = 2V$

$\therefore \Delta V = V_L - V_R = 8 - 2 = 6V$

b)

$R = \frac{1}{1/9 + 1/6} = 3.6\,\Omega$

$RC = 3.6 \times 10^{-6}$

$\exp(-t/RC) = \frac{1}{10} \Rightarrow$

$t = RC \ln 10 = 8.29\,\mu s$

7. Three 60.0-W, 120-V lightbulbs are connected across a 120-V power source, as shown in Figure P28.63. Find (a) the total power delivered to the three bulbs and (b) the voltage across each. Assume that the resistance of each bulb conforms to Ohm's law (even though in reality the resistance increases markedly with current).

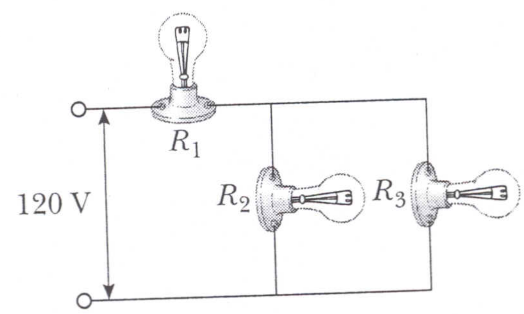


Figure P28.63

a) $R = \frac{\Delta V^2}{P} = \frac{(120)^2}{60} = 240\,\Omega$ for each

$R_{eq} = R_1 + \left(\frac{1}{1/R_2 + 1/R_3}\right) = 240 + 120 = 360\,\Omega$

Total Power dissipated $P = \frac{\Delta V^2}{R} = \frac{(120)^2}{360} = \boxed{40\,W}$

b) $P = I^2 R_{eq} \Rightarrow I = \sqrt{\frac{P}{R_{eq}}} = \sqrt{\frac{40}{360}} = \frac{1}{3}\,A$

$\Delta V_1 = I R_1 = \frac{1}{3} * 240 = \boxed{80\,V}$

$\Delta V_{23} = I R_{23} = \frac{1}{3} * \frac{1}{1/240 + 1/240} = \boxed{40\,V}$

8. A sinusoidal voltage $\Delta v(t) = (40.0\text{ V})\sin(100t)$ is applied to a series *RLC* circuit with $L = 160$ mH, $C = 99.0\ \mu$F, and $R = 68.0\ \Omega$. (a) What is the impedance of the circuit? (b) What is the maximum current? (c) Determine the numerical values for I_{max}, ω, and ϕ in the equation $i(t) = I_{max}\sin(\omega t - \phi)$.

a) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(68)^2 + (16 - 101)^2} = 109\ \Omega$

$X_L = L\omega = 100 \times 0.16 = 16\ \Omega$ (1)

$X_C = \frac{1}{C\omega} = \frac{1}{99\times10^{-6}\times100} = 101\ \Omega$

b) $I_{MAX} = \frac{\Delta V_{MAX}}{Z} = \frac{40}{109} = 0.367\text{ A}$

c) $\tan\phi = \frac{X_L - X_C}{R} = \frac{16 - 101}{68} = -1.25$

$\therefore \phi = -0.896\text{ Rad or } -51.3^\circ$

$I_{MAX} = 0.367\text{ A}$

$\omega = 100\text{ Rad/s}$

$\phi = -51.3^\circ$

(2)